term, all three terms of Eq. (10) contain both self-induced and interference effects.

## Discussion

The value of $\sigma$ is tabulated and presented graphically in Ref. 10 (pp. 217,218); it is reproduced here along with the new constant $\sigma_*$ in Fig. 2. $\sigma_*$ reflects the extent to which the momentum imparted to a small mass of air by the canard may be redistributed by the wing over a larger mass of air. Thus, when $h=0$, the wing is capable of redistributing the momentum over its entire span so that the minimum induced drag (regardless of canard load) is just that which would be produced by a single elliptically loaded wing carrying the total lift. This explains the interesting result that for many cases the interference terms are beneficial and the induced drag is actually lower than if the two wings were infinitely far apart. Equation (1), to the contrary, suggests that induced drag decreases with larger vertical gap whenever $\bar{L}>0$.

The ratio of the system's minimum induced drag to that of a monoplane with the span of the wing and carrying the same total lift is

$$\frac{1}{e}=\frac{1+2\sigma\bar{L}/\bar{b}+\sigma_*\bar{L}^2/\bar{b}^2}{(1+\bar{L})^2} \tag{13}$$

Since $\sigma_* = 1$ when the wing is elliptically loaded, the minimum induced drag given by Eq. (10) may differ significantly from that given by Eq. (1). If, for example, $2h/b_c=0.2$, $L_c/L_w=0.3$, and $b_c/b_w=0.4$, Eq. (13) gives $e=0.885$, while, if the surfaces were elliptically loaded, $e=0.802$. Differences become larger for smaller gaps and larger span loading ratios, $\bar{L}/\bar{b}$. Thus, although Eq. (13) may be used for estimating the induced drag of conventional, aft-tail configurations, it is especially useful for canard configurations which, because of stability and trim constraints, generally require larger values of $\bar{L}/\bar{b}$.

Figure 2 illustrates the rapid variation of $\sigma_*$ for small vertical gaps. This decrease in $\sigma_*$ reflects the substantial reduction in induced drag compared with the elliptically loaded case. Because of this sensitivity, however, the theoretical result that $e=1$ and is independent of $\bar{L}$ when no vertical gap is present is not achievable in practical cases where $h$ does not vanish completely.

## References

[1] Prandtl, L., "Induced Drag of Multiplanes," NACA TN-182, March 1924.

[2] Laitone, E., "Positive Tail Loads for Minimum Induced Drag of Subsonic Aircraft," *Journal of Aircraft*, Vol. 15, Dec. 1978, pp. 837-842.

[3] Wolkovitch, J., "Subsonic V/STOL Configurations with Tandem Wings," *Journal of Aircraft*, Vol. 16, Sept. 1979, pp. 605-611.

[4] McLaughlin, M., "Calculations, and Comparison with an Ideal Minimum of Trimmed Drag for Conventional and Canard Configurations Having Various Levels of Static Stability," NASA TN-D-8391, 1977.

[5] Sachs, G., "Minimum Trimmed Drag and Optimum C.G. Position," *Journal of Aircraft*, Vol. 15, Aug. 1978, pp. 456-459.

[6] Munk, M., "Minimum Induced Drag of Airfoils," NACA Report 121, 1921.

[7] Feistal, T., Corsiglia, V., and Levin, D., "Wind-Tunnel Measurements of Wing-Canard Interference and a Comparison with Various Theories," SAE Technical Paper 810575, April 1981.

[8] Reid, E.G., *Applied Wing Theory*, McGraw-Hill, New York, 1932.

[9] Laitone, E., "Prandtl's Biplane Theory Applied to Canard and Tandem Aircraft," *Journal of Aircraft*, Vol. 17, April 1980, pp. 233-237.

[10] von Kármán, T. and Burgers, J.M., "General Aerodynamic Theory—Perfect Fluids," Vol. II of *Aerodynamic Theory*, edited by W.F. Durand, Dover Edition, New York, 1963.

AIAA 82-4218

# Estimation of the Number of In-Flight Aircraft on Instrument Flight Rules

Norman Meyerhoff* and Jeffrey Garlitz†
*U.S. Department of Transportation, Cambridge, Mass.*

## Introduction

MATHEMATICAL models to estimate the instantaneous aircount (IAC) of aircraft on instrument flight rules (IFR) over the United States have been developed. IAC is required to help plan for new or augmented air traffic control (ATC) computers which process information on these aircraft in real-time.

An economical method to estimate IAC is required because it is prohibitively costly to process radar target reports, which, prior to this research, had been the standard way to count aircraft.[1] IAC, which is defined to be the number of in-flight aircraft at any instant of time in any single day, are estimated by new models which convert (nonradar) aggregate data on landings and takeoffs to IAC.

The IAC of IFR aircraft are estimated for each air route traffic control center (ARTCC, or simply center), of which there are 20 covering the continental airspace. The IAC are also forecast,[2] although this Note will be limited to a summary of a pilot study to compare current IAC predictions with field data. IFR aircraft consist of scheduled aircraft (air carriers and air taxis) and unscheduled aircraft [general aviation (GA) aircraft].

## Description of Models

The principal input to the scheduled model is the Official Airline Guide (OAG) tape, produced by the R.H. Donnelley Corporation. For each flight in the U.S., a great circle path is projected between the origin and destination to compute the location of an aircraft during specified intervals (in distance) of its flight. The aircount for a particular ARTCC and time of day is computed as follows. First, the scheduled departure and arrival times of each flight are examined to determine that the aircraft is in flight at the specified time of day (i.e., the time of day occurs between the departure and arrival times). Once it is established that the aircraft is in flight, the location of the flight at the particular time is computed, and if the aircraft's position (latitude, longitude) is determined to be over the relevant ARTCC, the aircount is incremented. Otherwise, the model examines the next flight from the OAG data using the same criteria. The model accesses a data base containing the boundaries of the ARTCCs, in the form of latitude and longitude, to determine which ARTCC the aircraft is passing over at a particular time of its flight.

There does not exist one easily accessible source of information for unscheduled aircraft operations. This would greatly complicate the problem were it not for the fact that the number of GA aircraft leaving an ARTCC is about equal to the number arriving, which means that only the number of IFR GA departures need be known to estimate the in-flight

Received Feb. 12, 1982. This paper is declared a work of the U.S. Government and therefore is in the public domain.

*Operations Research Analyst and Project Leader, Office of Air and Marine Systems, Operations Analysis Branch, U.S. Department of Transportation.

†Operations Research Analyst, Office of Air and Marine Systems, Operations Analysis Branch, U.S. Department of Transportation.

AUTHORS' NOTE: The views expressed in this Note are those of the authors but not necessarily those of the Department of Transportation or Federal Aviation Administration.

**Table 1 Steps leading to computation of probability of a departure in each time interval for Indianapolis ARTCC, April 21 and 28, 1978**

| | Number of flight service station departures | | | | | |
|---|---|---|---|---|---|---|
| | April 21, 1978 | | April 28, 1978 | | Total | Probability of |
| Time (LST)[a] | LOU[b] | LUK[c] | LOU | LUK | departures[d] | a departure[e] |
| 0700-0759 | 5 | 7 | 2 | 5 | 19 | 0.059 |
| 0800-0859 | 9 | 3 | 6 | 6 | 24 | 0.075 |
| 0900-0959 | 6 | 6 | 4 | 1 | 17 | 0.053 |
| 1000-1059 | 5 | 6 | 1 | 4 | 16 | 0.050 |
| 1100-1159 | 4 | 9 | 3 | 3 | 19 | 0.059 |
| 1200-1259 | 11 | 7 | 2 | 3 | 23 | 0.071 |
| 0100-0159 | 7 | 4 | 2 | 4 | 17 | 0.053 |
| 0200-0259 | 13 | 7 | 0 | 1 | 21 | 0.065 |
| 0300-0359 | 17 | 10 | 2 | 3 | 32 | 0.099 |
| 0400-0459 | 21 | 8 | 2 | 4 | 35 | 0.109 |
| 0500-0559 | 17 | 5 | 1 | 3 | 26 | 0.081 |
| 0600-0659 | 9 | 11 | 2 | 1 | 23 | 0.071 |
| 0700-0759 | 4 | 4 | 3 | 2 | 13 | 0.040 |

[a]These local times represent 7:00 a.m.-8:00 p.m. [b]Louisville flight service station. [c]Cincinnati flight service station. [d]For each hour, sum of all departures from Louisville and Cincinnati FSSs for April 21 and April 28, 1978. [e]Ratio of total departures in each time interval to total daily number of departures from LOU and LUK, both dates. The latter number is 322.

**Table 2 Steps leading to computation of flight time distribution of IFR general aviation aircraft for Indianapolis ARTCC, April 21 and 28, 1978**

| | Number of flight service station departures | | | | | |
|---|---|---|---|---|---|---|
| | April 21, 1978 | | April 28, 1978 | | Total | Probability of |
| Flight time, h | LOU[a] | LUK[b] | LOU | LUK | departures[c] | flight time[d] |
| 0.00-1.49 | 8 | 23 | 1 | 5 | 37 | 0.125 |
| 1.50-1.99 | 47 | 35 | 11 | 10 | 103 | 0.319 |
| 1.00-0.49 | 39 | 16 | 10 | 10 | 75 | 0.226 |
| 1.50-1.99 | 25 | 12 | 5 | 9 | 51 | 0.160 |
| 2.00-2.49 | 13 | 6 | 5 | 9 | 33 | 0.097 |
| 2.50-2.99 | 5 | 4 | 1 | 1 | 11 | 0.035 |
| 3.00-3.49 | 2 | 0 | 1 | 1 | 4 | 0.010 |
| 3.50-3.99 | 1 | 0 | 0 | 0 | 1 | 0.003 |

[a]Louisville flight service station. [b]Cincinnati flight service station. [c]For each flight time interval, sum of all departures from Louisville and Cincinnati FSSs for April 21 and 28, 1978. [d]Ratio of total departures to total daily number of departures from LOU and LUK, both dates. The latter number is 322.

aircount.[3] This count is computed using the following equation:

$$I(t) = \sum_{t'<t} DP(t')f(t-t') \tag{1}$$

where $I$ is the estimated ARTCC aircount of unscheduled aircraft at time $t$; $P$ is the probability of an IFR GA aircraft departure per unit time at $t'$; $f$ is the probability it will fly for a time interval $(t-t')$; and $D$ is the daily number of IFR GA departures from each ARTCC, recorded on FAA standard form 7230-14. GA flight plans (FAA form 7233-3) were sampled from 39 flight service stations (FSS) throughout the U.S. in April and May of 1978.[3] The collected data constitute about a 15% sample of all FSSs. The recorded departure times and estimated enroute flight times on the IFR flight plans were used to estimate $P$ and $f$ for the sampled FSSs.

Table 1 illustrates the procedure for using data from several sampled FSSs in Indianapolis ARTCC to compute the "Probability of a departure," $P$, in Eq. (1), for all IFR GA in Indianapolis ARTCC. The probability is computed using data from two FSSs on two successive Fridays in 1978. The two FSSs, Cincinnati and Louisville, are typical of the other FSSs in Indianapolis ARTCC.

The time distribution, $f$, is computed as follows. The recorded flight times are sampled in half-hour intervals, as shown in Table 2. (This is the minimum sampling frame consistent with adequate sample size.) Using the "Total departures" column, the flight time probability distribution is estimated in the last column.

The probabilities are estimated for all IFR GA in the ARTCC by sampling a few representative FSSs in the ARTCC, and then using statistical inference which assumes that the probabilities are the same for unsampled FSSs in the ARTCC. Usually, the probabilities are representative of the areas from which they are selected. They are similar for different FSSs in an ARTCC and are also similar for FSSs in different ARTCCs. The correlation appears strongest for aircraft in the same geographical region. For example, the combined probabilities for Chicago and Fort Wayne FSSs (both are in the Chicago ARTCC) are the same (10% difference or less) as the combined probabilities for Cincinnati and Louisville FSSs, both of which are in the neighboring Indianapolis ARTCC. This approximate invariance is a reflection of the fact that general aviation pilots' habits and the types of aircraft they fly are similar. Other traffic "constants," or "profiles," have been studied elsewhere and applied to other air traffic analyses.[3]

If the traffic profile for a specific ARTCC were unknown, a canned profile from a surrogate ARTCC could be used, because of regional similarities. For example, Eq. (1) was used to estimate the aircounts of unscheduled aircraft over Cleveland center. FSS data were not available from Cleveland center to compute $P$ and $f$ in the equation. However, similar data were available from Indianapolis center, and these data were used to compute $P$ and $f$ (see Tables 1 and 2), and subsequently $I(t)$ for Cleveland center. $D$ was extracted from Cleveland data (FAA form 7230-14).

The use of canned profiles, rather than the use of individual departure and arrival times of every aircraft to determine the

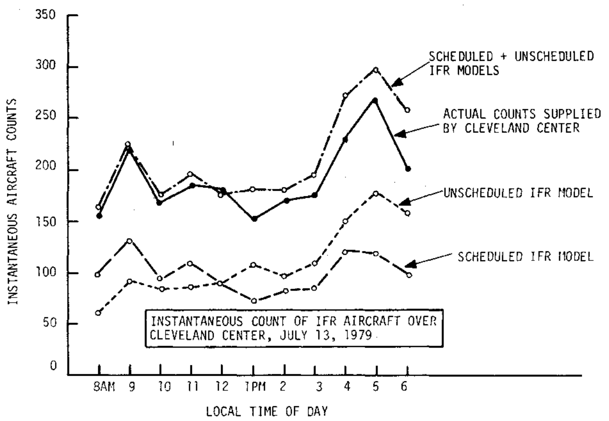


**Fig. 1 Instantaneous count of IFR aircraft over Cleveland center, July 13, 1979.**

number of aircraft in the air, allows for the development of an efficient model. The model is efficient in the sense that it is unnecessary to continuously collect and automate a potentially large amount of data, thereby greatly reducing computer costs incurred in processing those data. Additional air traffic analyses are also facilitated.

## Comparison of Model Predictions with Field Data

On July 13 (Friday), 1979, Cleveland center recorded the number of actively controlled aircraft as a function of time of day. (This recording also included a small number of controlled non-IFR aircraft.) Cleveland center, which controls the airspace over parts of Ohio, Michigan, New York, Pennsylvania, and West Virginia, is one of the busiest centers. The sum of the models' predicted counts of scheduled and unscheduled aircraft was compared with the recorded counts for the above data for the times, 8 a.m.-6 p.m., on the hour.

At the time the comparisons were made, unscheduled aircounts could be predicted for any day in April 1978. Therefore seasonal adjustment factors for GA activity based on historical data[4] were applied to these counts, so that they would be representative of counts on a typical Friday in July 1979.

Figure 1 shows predicted IFR counts and actual counts over Cleveland center. (Data points are connected by lines as a visual aid.) Similar comparisons were made for Houston, Kansas City, and New York centers (one day for each center in the summer of 1979, although the day of the week was different in each case). All comparisons between predicted and measured aircounts show good agreement in terms of small numerical differences in the amplitudes and also similar harmonic content. The largest difference occurs at the peaks and is about 10%. This agreement is sufficient for system planning purposes.[3] It is planned to extend these comparisons to other centers for additional days and seasons of the year, and to quantify the (statistical) differences between measured and predicted counts.

## Acknowledgment

This work was supported by the Federal Aviation Administration.

## References

[1] Meyerhoff, N., "Air Traffic Control System Measures and Data," in *Air Traffic Control Association Proceedings*, Air Traffic Control Association, Arlington, Va., Oct. 1979, pp. 172-180.

[2] "IFR Aircraft Handled, FY1980-1991," FAA Report No. FAA-AVP-80-4, 1980.

[3] Meyerhoff, N., "Estimation of the Instantaneous Aircount of General Aviation Aircraft Flying Under Visual Flight Rules," *Journal of Transportation Research*, Vol. 16A, No. 3, May 1982, pp. 225-241.

[4] "FAA Aviation Forecasts Fiscal Year 1979-1990," FAA Report No. FAA-AVP-78-11, 1978, p. 82.

**AIAA 82-4219**

# Unrestrained Aeroelastic Divergence in a Dynamic Stability Analysis

William P. Rodden* and E. Dean Bellinger†
*The MacNeal-Schwendler Corporation, Los Angeles, Calif.*

A QUASISTATIC formulation of the divergence problem for a vehicle free to plunge is given in Ref. 1 and has been generalized to multiple rigid body degrees of freedom in Ref. 2. The restrained divergence problem has been investigated by two dynamic stability methods in Ref. 3: the transient method of Ref. 4 and the British flutter method.[5-7] It is the purpose of this Note to investigate unrestrained divergence by the dynamic stability methods of Ref. 3 in order to determine the validity and accuracy of the quasistatic approximations of Refs. 1 and 2.

The first two examples of Ref. 3 are reconsidered here with the bending and torsion springs attached to a mass (fuselage), instead of to ground, free to move vertically. The characteristics of the two examples are the same except for the center of gravity location: in example 1 it is at 37% chord, and in example 2 it is at 45% chord. Both examples have the aerodynamic center at 25% chord ($\xi = 0.25$ in Ref. 4) and the elastic axis at 40% chord. The remaining parameters for the analysis at sea level include a chord of 6.0 ft, a radius of gyration of 1.5 ft about the elastic axis, a mass ratio $\mu = 20.0$, the uncoupled bending and torsion frequencies of 10.0 rad/s and 25.0 rad/s, respectively, and equal structural damping coefficients $g = 0.03$ in both modes. The lift curve slope is the theoretical incompressible value of $2\pi$ and the downwash is matched at the ¾-chord location ($r = 1.0$ in Ref. 4). The transient aerodynamic constants are $\alpha_1 = 0.165$, $\alpha_2 = 0.335$, $\beta_1 = 0.041$, $\beta_2 = 0.320$. In addition, the "fuselage" mass is assumed to be equal to the airfoil mass so that the mass ratio of Ref. 2 is $r = 0.50$.

The results of the transient solution for example 1 (with the forward center of gravity at 37%) are shown by the curves in Fig. 1. Divergence (from the aerodynamic lag root corresponding to $\beta_1 = 0.041$) occurs at 232.1 ft/s and flutter (from the torsion root) occurs at 294.3 ft/s. The stable solution for the second lag root (corresponding to $\beta_2 = 0.320$) is not shown. In contrast with Ref. 3, in which divergence was nonoscillatory, we see in Fig. 1 that the unrestrained divergence is *oscillatory* with a frequency of 1.189 Hz. We seem to have a semantic problem now. An unrestrained vehicle does not diverge statically but dynamically: shall we call this instability divergence or flutter? The oscillatory characteristic of unrestrained divergence has been calculated before in studies of the oblique-wing aircraft[8-11] but in those studies it was described as low frequency flutter. The present authors will continue to use the terminology of "dynamic divergence" because the instability finds its origin in a tendency to static divergence. The terminology "body freedom flutter" has also been used to describe dynamic divergence of forward-swept-wing aircraft,[12] but that does not seem appropriate since Gaukroger[13] has used those terms to describe a coupled body-pitch/wing-bending flutter in which the body has a relatively low value of pitching moment of inertia; the examples here have no rigid body degree of freedom in pitch.

Received May 7, 1982. 

*Consultant. Associate Fellow AIAA.
†Staff Scientist.